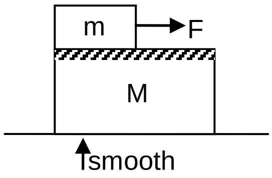
PART – I (PHYSICS)

SECTION – A (Total Marks : 21)

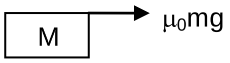
1. A plank having mass M is placed on smooth horizontal surface. Block of mass m is placed on it coefficient of friction between block and plane is $\mu_0 + kx$, where k is constant and x is relative displacement of block w.r.t. plank. A force F is applied on block where F = at, where a = 10; t is in second. Find t_0 when relative motion will occur between block and plank (use g = 10 m/s^2).



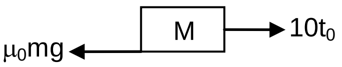
(A) $\mu_0 M + \frac{\mu_0 M^2}{m}$ (B) $\mu_0 m + \frac{\mu_0 M^2}{m}$ (C) $\mu_0 m + \frac{\mu_0 m^2}{M}$ (D) $\mu_0 M + \frac{\mu_0 m^2}{M}$

Ans. C

Sol. Let at time t_0 relative motion will occur



$\mu_0 mg = Ma$ ……..(1)



$10t_0 - \mu_0 mg = ma$…………..(2)

From (1) and (2);

$$t_0 = \mu_0 m + \frac{\mu_0 m^2}{M}$$

2. A short dipole $\vec{p}$ is at a distance r from a point charge q and oriented so that $\vec{p}$ makes angle θ with position vector $\vec{r}$ from q to $\vec{p}$. If θ is variable, then what is ratio of minimum and maximum magnitudes of force on the dipole:

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) 0 (D) 1

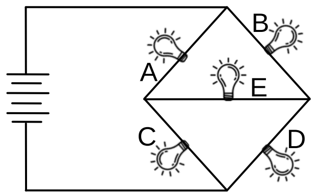
Ans. A

Sol. $F_{max} = \frac{2kpq}{r^3}$

$F_{min} = \frac{kpq}{r^3}$

$\frac{F_{min}}{F_{max}} = \frac{1}{2}$

3. Five bulbs A, B, C, D and E of power ratings 50W, 75 W, 40 W, 60 W, and 100 W, at household supply are connected as shown in the figure. Which bulb will consume least power (Neglect variation of resistance with temperature)

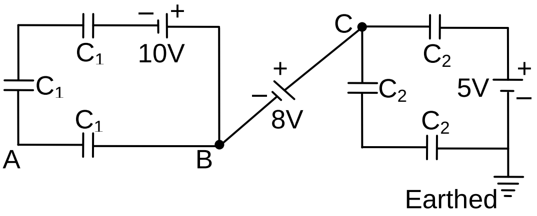


(A) B (B) C
(C) D (D) E

Ans. D

Sol. Since wheat stone bridge is balanced so power consumed by the bulb E is zero.

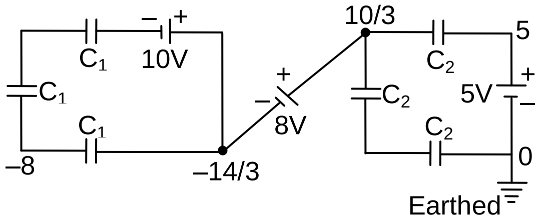
4. In the shown circuit diagram all capacitors are initially uncharged and cells are ideal. If $C_1 = 1\ \mu F$, $C_2 = 2\ \mu F$ and potential of earth is taken zero. Choose the correct option.



(A) Potential of point A is (4/3) volts
(B) Potential of point B is (–10/3) volts
(C) Potential of point C is (10/3) volts
(D) Potential of point A is (–4/3) volts

Ans. C

Sol.



$$5 - V_C = \frac{5}{3}$$

$$V_C = 5 - \frac{5}{3} = \frac{10}{3}$$

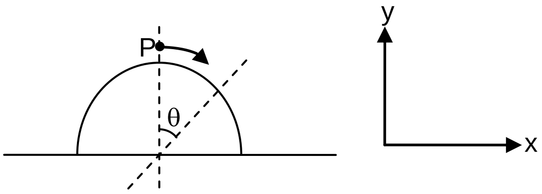
$$\frac{10}{3} - V_B = 8$$

$$V_B = \frac{-14}{3}$$

$$\frac{-14}{3} - V_A = \frac{10}{3}$$

$$V_A = \frac{-24}{3} = -8$$

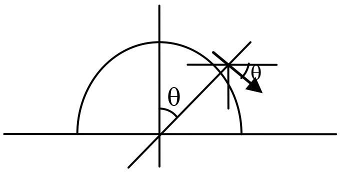
5. A particle P moves along the given circular path such that its x component of velocity always remains constant as V_0. Let V be the velocity of particle a function of θ. Then the choose the correct option.



(A) $V = V_0 \cos\theta$
(B) $V = V_0 \sec\theta$
(C) $V = \frac{2V_0}{\cos\theta}$
(D) $V = \frac{V_0}{\sec\theta}$

Ans. B

Sol.



$V \cos\theta = V_0$

$$V = \frac{V_0}{\cos\theta} = V_0 \sec\theta$$

6. Astronomers observe two separate solar systems, each consisting of a planet orbiting a sun. The two orbits are circular and have the same radius R. It is determined that the planets have angular momenta of the same magnitude L about their suns, and that the orbital periods are in the ratio of three to one; i.e., $T_1 = 3T_2$. The ratio m_1/m_2 of the masses of the two planets is

(A) 1 (B) $\sqrt{3}$ (C) 2 (D) 3

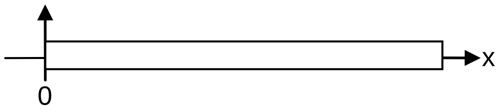
Ans. D

Sol. $L = mVR \qquad V = \frac{2\pi R}{T}$

$$L = mR\left(\frac{2\pi R}{T}\right) = \frac{2\pi m R^2}{T}$$

$$\frac{L_1}{L_2} = \frac{m_1}{T_1}\frac{T_2}{m_2} \Rightarrow \frac{m_1}{m_2} = \frac{T_1}{T_2} = 3$$

7. A rod of length L and mass M is placed along the x-axis with one end at the origin, as shown in the figure above. The rod has linear mass density $\lambda = \frac{2M}{L^2}x$, where x is the distance from the origin. Which of the following gives the x-coordinates of the rod's centre of mass ?

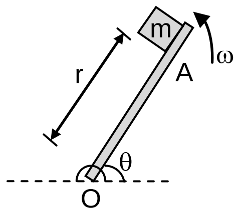


(A) $\frac{2}{3}L$ (B) $\frac{1}{4}L$ (C) $\frac{1}{3}L$ (D) $\frac{1}{2}L$

Ans. A

Sol. $$X_{cm} = \frac{1}{M}\int x\,dm = \frac{1}{M}\int_0^L x\left(\frac{2m}{L^2}x\right)dx = \frac{2L}{3}$$

8. The member OA rotates about a horizontal axis through O with a constant counter clockwise velocity ω = 3 rad/sec. As it passes the position θ = 0, a small mass m is placed upon it at a radial distance r = 0.5 m. If the mass it at a radial distance r = 0.5 m. If the mass is observed to slip at $\theta = 37^o$, the coefficient of friction between the mass & the member is :



(A) $\frac{3}{16}$ (B) $\frac{9}{16}$

(C) $\frac{4}{9}$ (D) $\frac{5}{9}$

Ans. A

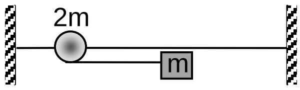
Sol. As the mass is at the verge of slipping

$\therefore$ mg sin37 – μ mg cos37 = $m\omega^2 r$

$6 - 8\mu = 4.5$

$\therefore \mu = \frac{3}{16}$

9. A bead can slide on a smooth straight wire and a particle of mass m attached to the bead by a light string of length L. The particle is held in contact with the wire and with the string taut and is then let fall. If the bead has mass 2 m then when the string makes an angle θ with the wire, the bead will have slipped a distance:



(A) $L(1-\cos\theta)$ (B) $(L/2)(1-\cos\theta)$

(C) $(L/3)(1-\cos\theta)$ (D) $(L/6)(1-\cos\theta)$

Ans. **C**

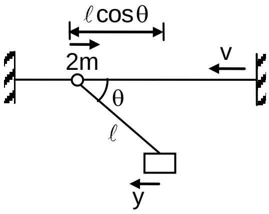
Sol. Since the centre of mass will not move

$2mx = my \Rightarrow \quad y = 2x$

and $x + y = \ell - \ell\cos\theta = l(1-\cos\theta)$

$3x = l(1 - \cos\theta)$

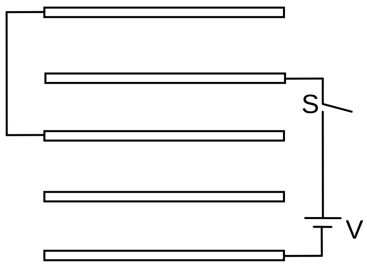
$\Rightarrow x = \dfrac{\ell(1-\cos\theta)}{3}$ or $x = \dfrac{\ell(1+\cos\theta)}{3}$



If angle made is (–) from the left side

10. 5 identical thin conducting plates are kept parallel to each other such that separation between each plate is d. Area of each plate is A. Plates are arranged as shown in figure. All wires are conducting.

Switch S is closed, find the work done by battery. ($A >>> d^2$)

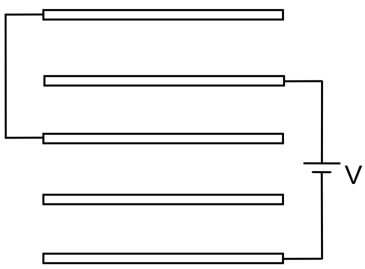


(A) $\dfrac{\epsilon_0 AV^2}{5d}$ (B) $\dfrac{2\epsilon_0 AV^2}{5d}$

(C) $\dfrac{3\epsilon_0 AV^2}{2d}$ (D) $\dfrac{2\epsilon_0 AV^2}{d}$

Ans. **B**

Sol.



(1) $\dfrac{q_1}{A\epsilon_0}d - \left(\dfrac{q_0 - q_1}{A\epsilon_0}\right)d = 0$

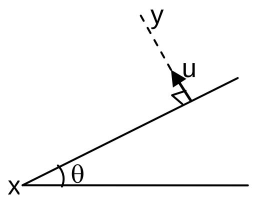
$\Rightarrow q_1 - q_0 + q_1 = 0 \Rightarrow q_1 = \dfrac{q_0}{2}$

(2) $\dfrac{q_1 d}{A\epsilon_0} + \dfrac{q_0 d}{A\epsilon_0} + \dfrac{q_0 d}{A\epsilon_0} = V$

$\Rightarrow \dfrac{d}{\epsilon_0 A}\left(\dfrac{q_0}{2} + q_0 + q_0\right) = V$

$$q_0 = \frac{2 \epsilon_0 AV}{5d}$$

11. A ball is projected perpendicularly from aninclined plane of angle θ, with speed 'u' as shown. The time after which the projectile is making angle 45^o with the inclined plane is :



(A) $\frac{u}{g\sin\theta}$ (B) $\frac{u}{g\cos\theta}$

(C) $\frac{u}{g\{\sin\theta + \cos\theta\}}$ (D) $\frac{u}{g\{\sin\theta - \cos\theta\}}$

Ans. C

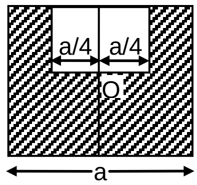
Sol. $V_y = u - g\cos\theta.t$

$V_x = g\sin\theta.t$

$$\tan\alpha = \frac{V_y}{V_x}$$

$$\tan 45^o = \frac{u - gt\cos\theta}{gt\sin\theta} \Rightarrow t = \frac{u}{g\{\sin\theta + \cos\theta\}}.$$

12. A square plate of edge a/2 is cut out from a uniform square plate of edge 'a' as shown in figure. The mass of the remaining portion is M. The moment of inertia of the shaded portion about an axis passing through 'O' (centre of the square of side a) and perpendicular to plane of the plate is:



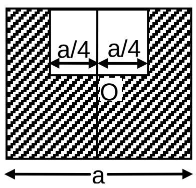
(A) $\frac{9}{64}Ma^2$ (B) $\frac{3}{16}Ma^2$

(C) $\frac{5}{12}Ma^2$ (D) $\frac{Ma^2}{6}$

Ans. B

Sol. Let m_1 = mass of the square plate of side 'a'

and m_2 = mass of the square of side 'a/2'



Then $m_1 = \sigma\left(\frac{a}{2}\right)^2 ; m_2 = \sigma(a)^2$

(σ being the areal density)

and $m_2 - m_1$ = M.

$$\Rightarrow I = \frac{m_2a^2}{6} - \left\{\frac{m_1(a/2)^2}{6} + m_1\left(\frac{a}{4}\right)^2\right\}$$

$$= \frac{\sigma\, a^4}{6} - \left\{\frac{\sigma\,(a/2)^4}{6} + \sigma\left(\frac{a}{2}\right)^2 \cdot \left(\frac{a}{4}\right)^2\right\}$$

$$= \sigma\, a^4\left\{\frac{1}{6} - \frac{1}{16\times6} - \frac{1}{4\times16}\right\}$$

$$= \sigma a^4 \left\{ \frac{(2\times16)-2-3}{16\times12} \right\}$$

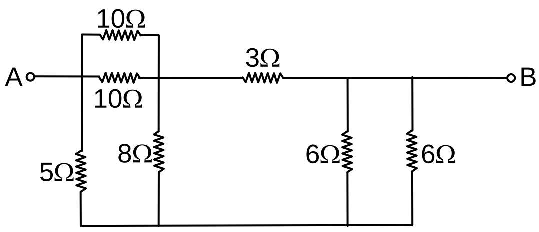
$$I = \sigma a^4 \left\{ \frac{27}{12\times16} \right\}$$

$$\text{Also; } M = \sigma\left(1-\frac{1}{4}\right)a^2 \Rightarrow \sigma = \frac{4}{3}\frac{M}{a^2}$$

$$\Rightarrow I = \left(\frac{4}{3}\frac{M}{a^2}\right).a^4\left\{\frac{27}{12\times16}\right\}$$

$$I = \frac{3Ma^2}{16}$$

13. Equivalent resistance between point A and B is

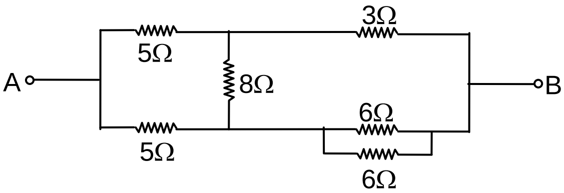


(A) 3Ω (B) 4Ω

(C) 4.5Ω (D) 5Ω

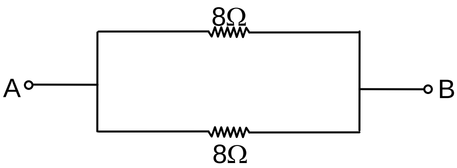
Ans. B

Sol.



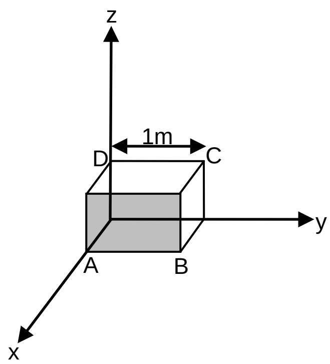
Which is a balanced wheat stone bridge

So



Req = 4Ω

14. In cubical volume of side 1 m, the electric field is $E = E_0\left(1+\frac{z}{a}\right)\hat{i} + E_0\left(1-\frac{z}{a}\right)\hat{j}$ where E_0 = 0.2 N/C and a = 1m. Cube has one vertex at origin and sides are parallel to co-ordinates axes. The flux coming out from the front shaded face ABCD will be



(A) $\frac{3}{5}N-m^2/C$ (B) $\frac{3}{10}N-m^2/C$

(C) $\frac{2}{5}N-m^2/C$ (D) 0

Ans. B

Sol. $\phi = \int \vec{E}.\vec{dA}$

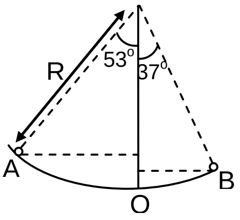
$= \int \vec{E}.(adz)\hat{i}$

$$\int_0^a E_0\left(1+\frac{z}{a}\right)adz$$

$$=E_0=\left(a^2+\frac{a^2}{2}\right)=\frac{3E_0a^2}{2}$$

$$\frac{3}{10}Nm^2/C$$

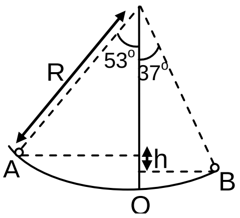
15. A section of fixed smooth circular track of radius R in vertical plane is shown in figure. A small block is released from position A and it leaves the track at B. The radius of curvature of its trajectory when it reaches to highest point of path during its motion in air is :



(A) $\frac{R}{2}$
(B) $\frac{R}{4}$
(C) $\frac{2}{5}$
(D) $\frac{32}{125}R$.

Ans. D

Sol.



$$mg\left(R\cos 37^o - R\cos 53^o\right)=\frac{1}{2}mv^2$$

$$mgR\left(\frac{4}{5}-\frac{3}{5}\right)=\frac{1}{2}mv^2$$

$$gR\left(\frac{1}{5}\right)=\frac{V^2}{2}$$

$$V=\sqrt{\frac{2}{5}gR}$$

ROC at H_{max}

$$ROC=\frac{V_1^2}{a_1}$$

$$=\frac{\left(V\cos 37^o\right)^2}{g}=\frac{\frac{2}{5}gR\times\frac{16}{25}}{g}\Rightarrow\frac{32}{125}R$$

16. A convex lens has mean focal length of 20 cm. The dispersive power of the material of the lens is 0.02. Then the longitudinal chromatic aberration for an object at infinity is

(A) 0.20 cm
(B) 0.40 cm
(C) 10^{-3} cm
(D) 10^{3} cm

Ans. B

Sol. $\frac{1}{f}=\left(\mu_y-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

$$\Rightarrow\frac{1}{f}=\left(\mu-1\right)\left(k\right)$$

Differentiating –

$$-\frac{1}{f_2}df=\left(d\mu\right)\left(k\right)$$

$$df = -(d\mu)k\left(f^2\right)$$

$$df = \frac{-(d\mu)k}{(\mu-1)(k)}(f) \qquad \frac{1}{f} = (\mu-1)(k)$$

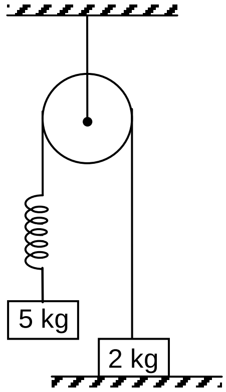
$$= -\omega f. \Rightarrow \frac{d\mu}{\mu-1} = \omega = -0.02 \times 20 = 0.40\text{cm}$$

17. Two moles of an ideal monoatomic gas occupies a volume V at 27^oC. The gas expands adiabatically to a volume 2V. Calculate (i) the final temperature of the gas and (ii) change in its internal energy.

(A) (i) 189 K (ii) 2.7 kJ
(B) (i) 195 K (ii) –2.7 kJ
(C) (i) 189 K (ii) –2.7 kJ
(D) (i) 195 K (ii) 2.7 kJ.

Sol. C

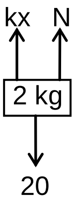
18. The system as shown in the figure is released from rest. The pulley, spring and string are ideal & friction is absent everywhere. The speed of 5 kg block when 2 kg block leaves the contact with ground is : (spring constant k = 40 N/m & g = 10 m/s^2)



(A) $\sqrt{2}\text{m/s}$
(B) $2\sqrt{2}\text{m/s}$
(C) 2 m/s
(D) $4\sqrt{2}\text{m/s}$

Ans. **B**

Sol. F.B.D of 2 kg block



When 2 kg block just leaves the contact

N = 0

$$kx + 0 = 20 \Rightarrow x = \frac{20}{40} = \frac{1}{2}\text{m}$$

Applying WET for the whole system

$w_g + w_{sf} = \Delta k$

$$\Rightarrow 50 \times \frac{1}{2} - \frac{1}{2}(40)\left[\left(\frac{1}{2}\right)^2 - 0\right] = \frac{1}{2}(5)[V^2 - 0]$$

$$\Rightarrow V = 2\sqrt{2}\text{m/s}$$

19. A block of mass 10kg is placed on rough inclined plane of variable angle θ and friction coefficient $\mu_s = \mu_k = 3/4$. When θ is 37^o net reaction force applied by inclined is $\vec{N}_1$ and when $\theta = 53^o$ net reaction force applied by inclined is $\vec{N}_2$, then $|\vec{N}_1| - |\vec{N}_2|$ is:

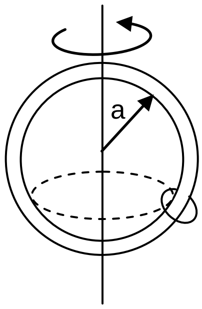
(A) 0
(B) 25 N
(C) –25 N
(D) –37 N

Ans. **B**

Sol. $Mg - \sqrt{1+\mu^2}Mg\cos\theta$

= 25 N.

20. A small bead of mass 'm' is threaded on a frictionless circular wire of radius 'a'. The circular wire frame is rotates about its vertical diameter as shown. (Assume acceleration due to gravity is g) The angular speed required if the bead is to be made to move in a horizontal circle of radius $\frac{a\sqrt{3}}{2}$ is:



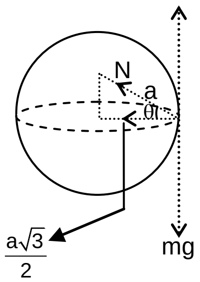
(A) $\left(\frac{g}{a}\right)^{1/2}$ (B) $\left(\frac{2g}{a}\right)^{1/2}$

(C) $\left(\frac{3g}{2a}\right)^{1/2}$ (D) $\left(\frac{3g}{a}\right)^{1/2}$

Ans. B

Sol. N sinθ = mg

$N\cos\theta = m\omega^2 \frac{a\sqrt{3}}{2}$



When friction in absent;

$\therefore \tan\theta = \frac{2g}{\omega^2 a\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\omega = \left(\frac{2g}{a}\right)^{\frac{1}{2}}$

21. Three similar cells, each of emf 2V and internal resistance r send the same current through an external resistance of 2Ω, when connected in series or in parallel. Then the magnitude of current flowing through the external resistance is :

(A) 0.75 A (B) 1 A (C) 1.5 A (D) zero

Ans. A

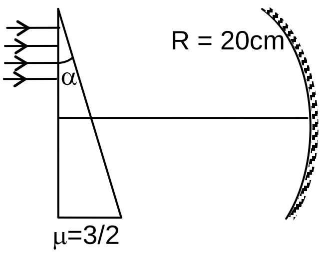
Sol. In series $I = \frac{3\times 2}{3r+2}$

In parallel $I = \frac{2}{r/3+2}$

but $\frac{6}{3r+2} = \frac{6}{r+6}$ $r = 2\Omega$

then $I = \frac{3\times 2}{3\times 2+2} = 0.75\text{Amp.}$

22. In the given figure a parallel beam of light is incident on the upper part a prism of angle 3.6° and R.I. 3/2. The light coming out of the prism falls on a concave mirror of radius of curvature 20 cm. The distance of the point (where the rays are focused after reflection from the mirror) from the principal axis is: [use π = 3.14]

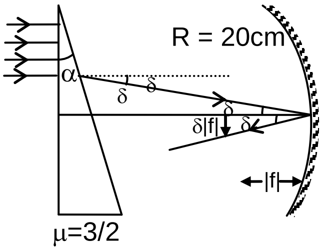


(A) 9 cm (B) 1.57 mm

(C) 3.14 mm (D) none of these

Ans. C

Sol. Deviation by prism $= 3.6^o\left(\frac{3}{2}-1\right) = 1.8^o$



R = 20 cm |f| = 10 cm

Image will form on focal plane

Distance of image from y–axis = |f| δ

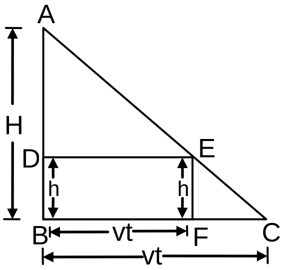
$= 100 \times \frac{1.8\pi}{180} \text{mm} = 3.14\text{mm}$

Integer Value Correct Type

23. A man of height 1.8 m walks away from a lamp at a height of 6m. If the man's speed is 3.5 m/sec. Find the speed in m/sec at which the tip of the shadow moves.

Ans. **5 m/sec**

Sol.



BF = Vt; BC = V't

from similar triangles.

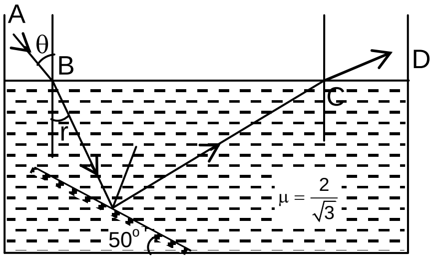
EFC and ABC

$\frac{FC}{BC} = \frac{EF}{AB} = \frac{h}{H}$

$\frac{FC}{BC} = \frac{EF}{AB} = \frac{h}{H} \Rightarrow \frac{V't - Vt}{V't} = \frac{h}{H}$

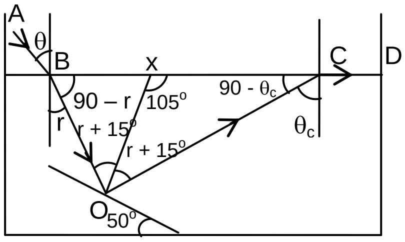
$\Rightarrow V' = \frac{HV}{H-h} = 5 \text{m/sec}$

24. An inclined plane mirror lies at the bottom of a large containing a liquid as shown in figure. Narrow monochromatic beam falls on surface of water at angle of incidence θ. Beam comes: out of the liquid at C. If maximum angle of incidence θ for which light comes out liquid is $\sin^{-1}\left(\frac{\sqrt{3}}{K}\right)$ then find value of K.



Ans. 3

Sol.



In triangle $\cos 105^0 + r + 15^0 + 90 - \theta_C = 180$

$r + 30^0 = \theta_C$

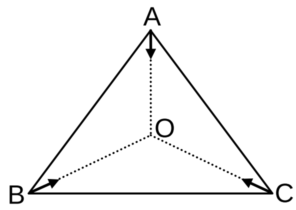
Then $\sin(r + 30^0) = \sin\theta_C = \frac{\sqrt{3}}{2}$

$r = 30^0$

Now, $\frac{\sin\theta}{\sin 30^0} = \frac{2}{\sqrt{3}} \Rightarrow \theta = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$\theta = \sin^{-1}\left(\frac{\sqrt{3}}{3}\right)$

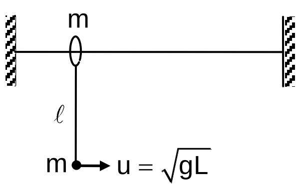
25. Three particles of equal masses are initially at the vertices of equilateral triangle of side $2\sqrt{3}m$ in horizontal plane. They start moving towards centriod O with equal speed 2 m/sec. After collision at O, A stops and C retraces its path with same speed. Distance between B and C just after one second of the collision is α m. Here α is an integer. Find α.



Ans. 4

Sol. Before collision, linear momentum of the system is zero. Therefore, after collision momentum of B will be equal and opposite to momentum of C (since $P_A = 0$)

26. In the figure shown a small ring of mass m and a small object of the same mass m are tied to the ends of light inextensible string of length $\ell = 4m$ The ring can slide along a fixed horizontal smooth rod. The string is initially vertical and the object is at its lowest position. Now a horizontal velocity $u = \sqrt{g\ell}$ is given to the object parallel to the rod. Find the maximum height (in metres) attained by the object from initial position.



Ans. 1

Sol. At the highest point velocity of object relative to the ring will become zero.

By momentum conservation in horizontal direction

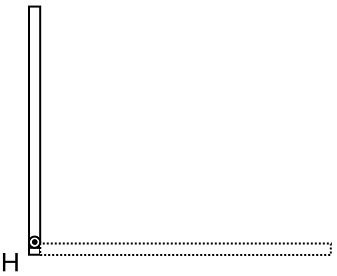
$mu = mv + mv \Rightarrow \quad v = u/2$

and by energy conservation

$$\frac{1}{2}mu^2 = \frac{1}{2}2m\left(\frac{u}{2}\right)^2 + mgh$$

$$\Rightarrow \frac{1}{4}mu^2 = mgh \qquad \Rightarrow \qquad h = \frac{u^2}{4g} = \frac{g\ell}{4g} = 1m$$

27. A thin rod of length 1 m is kept vertical and is hinged from the lower end 'H'. Its linear mass density veries with the distance (x) from the end 'H' as $\lambda = (12\ x)$ kg/m. where x is distance along rod from point H. The potential energy (in Joules) of the rod is 5N. Find N (at H the potential energy is zero) (g = 10 m/s^2)



Ans. 8

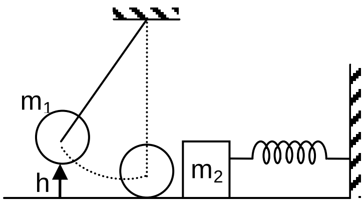
Sol. Lets take an element of dx width at a distance x from the lowest end.

$$dU = (dm)gx \int_{x=0}^{x=1} x^2 dx = 40J$$

$$dU = (12xdx)gx$$

U = 12 g

28. A pendulum bob 0.5 kg is raised to a height of 15 cm before it is released. At the bottom of its path, it makes a perfectly elastic collision with a 1 kg mass that is connected to the horizontal spring of spring constant 1.5 N/m. Maximum compression in spring is $\frac{280}{x}$cm. Find the value of x.



Ans. 3

Sol. $m_1gh = \frac{1}{2}m_1v_1^2 \Rightarrow v_1 = \sqrt{2gh}$

$$m_1v_1 = m_1v_1^{'} + m_2v_2 \qquad \textbf{............(i)}$$

$$1 = \frac{v_2 - v_1^{'}}{v_1} \Rightarrow v_1 = v_2 - v_1^{'}$$

From (1) & (2)

$$0.5v_1 = 0.5(v_2 - v_1) + 1v_2$$

$$v_2 = \frac{v_1}{1.5}$$

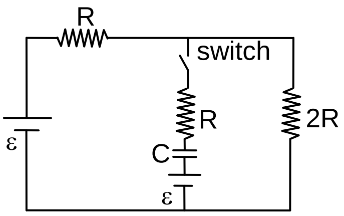
$$\frac{1}{2}m_1v_2^2 = \frac{1}{2}Kx^2$$

$$x^2 = \frac{m_2v_2^2}{K}$$

$$\Rightarrow x^2 = \sqrt{\frac{m_2}{K}} \times v_2 = \sqrt{\frac{1}{1.5}} \times \frac{\sqrt{2 \times 9.8 \times 1.5}}{1.5}$$

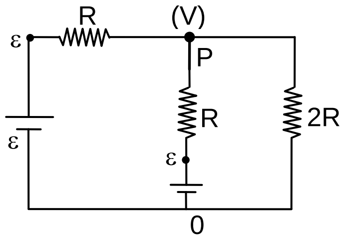
$$= \frac{2}{3}\sqrt{\frac{2 \times 9.8 \times 1.5}{1.5 \times 100}} = \frac{2 \times 14}{3 \times 10} = \frac{2.8}{3} \Rightarrow n = 3$$

29. Initially the capacitor was uncharged. Current in the branch having capacitor, just after switching on is $\frac{\varepsilon}{xR}$. Then x is:



Ans. 5

Sol. **Just after switching , the capacitor will act like a conducting wire. So effective circuit will be**

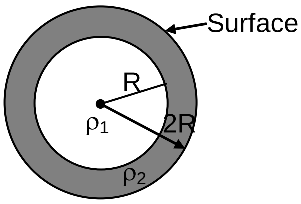


By applying KVL at point P $\frac{V-\varepsilon}{R}+\frac{V-\varepsilon}{R}+\frac{V}{2R}=0$

$V=\frac{4\varepsilon}{5}$

So, current in the branch containing capacitor $=\frac{\varepsilon}{5R}$

30. A planet is made of two materials of density ρ_1 and ρ_2 as shown in figure. The acceleration due to gravity at surface of planet is same as a depth 'R'. The ratio of $\frac{\rho_1}{\rho_2}$ is $\frac{x}{3}$. Find the value of x.



Ans. 7

Sol. $\frac{GM}{(2R)^2}=\frac{GM'}{R^2}$

$\frac{M}{4}=M'$

$\frac{4}{3}\pi R^3\rho_1+\frac{4}{3}\pi(8R^3-R^3)\rho_2=4\left(\frac{4}{3}\pi R^3\rho_1\right)$

$\rho_1+7\rho_2=4\rho_1$

$\frac{\rho_1}{\rho_2}=\frac{7}{3}$

PART – II (CHEMISTRY)

1. Select the incorrect statement among the following:
(A) For the phenomenon of adsorption $\Delta G < 0$.
(B) At high concentration of soap in water, soap behaves as associated colloid.
(C) At the equilibrium position in the process of adsorption $\Delta H = T\Delta S$.
(D) The term 'sorption' stands for adsorption.

Ans. **D**

Sol. The term sorption stands for both adsorption and absorption.

2. Which of the following statement(s) is/are correct ?
(I) An octahedral complex of zinc(II) ion has zero CFSE value.
(II) A solution of $[Ni(H_2O)_6]^{2+}$ is colored as the value of Δ_o for the H_2O complex fall in the visible region.
(III) The correct order for the wavelengths of absorption in the visible region for $[Ni(NO_2)_6]^{4-}$, $[Ni(NH_3)_6]^{2+}$, $[Ni(H_2O)_6]^{2+}$ will be in order $[Ni(NO_2)_6]^{4-} < [Ni(NH_3)_6]^{2+} < [Ni(H_2O)_6]^{2+}$
(A) I (B) I, II, III (C) I, II (D) II, III

Ans. **B**

Sol. **(I)** An octahedral complex of zinc (II) ion has d^{10} electronic configuration so has zero CFSE value.
(II) Paramagnetic complex are normally coloured complex.
(III) $\text{Wavelength} \propto \dfrac{1}{\Delta_o}$

$$\propto \frac{1}{\text{strength of ligand}}$$

3. A six co-ordinate complex has the formula $CoCl_3.5NH_3.H_2O$. Electrical conductance measurements indicate the presence of three ions in one formula unit. How money moles of AgCl will be precipitated with excess $AgNO_3$ solution with two mole of complex?
(A) 4 (B) 3 (C) 2 (D) 1

Ans. **A**

Sol. **III** **III**

$$[Co(NH_3)_5Cl]Cl_2.H_2O_{(aq)} \rightleftharpoons [Co(NH_3)_5Cl]^{2+}(aq) + 2Cl^-(aq)$$

$$2Ag^+ + 2Cl^- \rightarrow 2AgCl\downarrow (\text{white})$$

4. What volume of 2×10^{-4} M $Ba(OH)_2$ must be added to 300 mL of a 1×10^{-4}M HCl solution to get a solution in which the molarity of hydronium (H_3O^+) ions is 5×10^{-11} M?
(A) 375 mL (B) 300 mL (C) 225 mL (D) 450 mL

Ans. **D**

Sol. $$[OH] = \frac{K_w}{[H^+]} = \frac{10^{-14}}{5\times10^{-11}}$$

$$\frac{V\times2\times10^{-4}\times2-300\times1\times10^{4}}{300+V} = 2\times10^{-4}[OH]$$

V = 450 mL

5. Molar solubility of Hg_2I_2 in distilled water is ($K_{sp} = 4 \times 10^{-24}$) :
(A) 7.07×10^{-7} M (B) 10^{-8} M (C) 1.414×10^{-6} M (D) None of these

Ans. **B**

Sol. $$Hg_2I_2(s) \rightleftharpoons Hg_2^{2+}(aq) + 2I^-(aq)$$

$$K_{sp} = 4s^3$$

$$\therefore 4\times10^{-24} = 4s^3 \qquad \therefore s = 10^{-8}M$$

6. What is the molarity and molality of CH_3COOH solution that has density 1.2 gm/mL at 25^oC and contains 10% by weight of CH_3COOH?

(A) 2 M, 1.85 m (B) 0.5 M, 0.75 m (C) 3 M, 2 m (D) 2 M, 0.85 m

Ans. A

Sol. $M = \frac{(\%w/w)d \times 10}{Mol.wt.}$

$= \frac{10 \times 1.2 \times 10}{60} = 2M$

$m = \frac{M}{1000d - MM_2}$

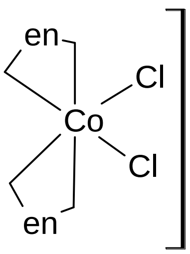
$= \frac{2}{1000d - MM_2}$

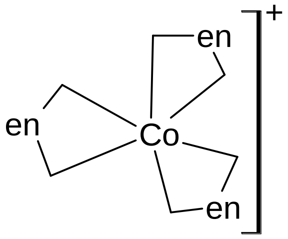
$= \frac{2}{1000 \times 1.2 - 2 \times 60} \times 1000$

= 1.85 m

7. Select the **INCORRECT** statement:

(A) Octahedral complex $[Rh(NO_2)(SCN)(en)_2]^+$ can exhibit optical isomerism.

(B)


and


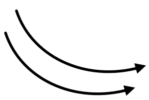
are optically active.

(C) If complex $[Ni(en)_3]^{2+}$ has 'x' number of chelate ring and 'y' number of stereoisomer then x + y will be 6.

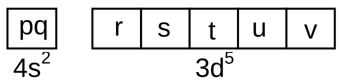
(D) The number of geometric isomers possible for the complex $[M(AB)a_2b_2]$ is 4. (Where AB $\rightarrow$ Unsymmetric bidentate ligand. a, b $\rightarrow$ monodentate ligands)

Ans. C

Sol (3) complex (1) $[Ni(en)_3]^{2+} \Rightarrow$ 3 chelate ring


2 optical isomer

0 G.I

8.


If the spin quantum number of 'p' and 'u' is same. The group of electrons (Given in the same bracket in options) having at least three quantum numbers same are:

(A) (pq) (rst) (tup) (B) (pr) (qs) (stu) (C) (pq) (rst) (tuv) (D) None of these

Ans. C

Sol.

pq	r	s	t	u	v
$4s^2$	$3d^5$				

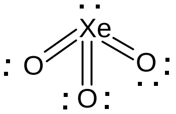
	$4s^2$	$3d^5$
n	4	3
ℓ	0	2s
m	0	-2, -1, 0, 1, 2
spin	↿⇂	↿ ↿ ↿ ↿ ↿

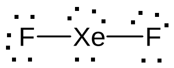
9. In which of the following species, each atom carries same number of lone pair of electrons on it?

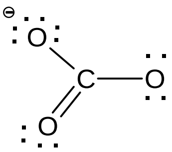
(A) XeO_3 (B) XeF_2 (C) CO_3^{2-} (D) O_3

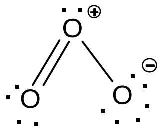
Ans. **B**

Sol.

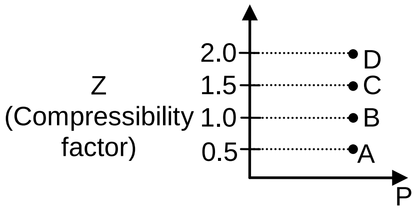
(1) 

(2)  all three atoms carry 3 lp each

(3) 

(4) 

10. Molar volume of an ideal gas is 0.45 dm^3/mol. The molar volume of air (assuming as real gas) under the same condition is 0.9 dm^3/mol. The point which corresponds to air in the given graph is:



(A) B (B) D (C) A (D) None of these

Ans. **B**

Sol. $z = \frac{0.9}{0.45} = 2 \quad z > 1$

11. The IUPAC name of $[Ni(NH_3)_4][NiCl_4]$ is :
(A) Tetrachloridonickel(II) tetraamminenickel(II)
(B) Tetraamminenickel(II) tetrachloridonickel(II)
(C) Tetraamminenickel(II) tetrachloridonickelate(II)
(D) Tetrachloridonickel(II) tetraamminenickelate(0)

Ans. **C**

Sol. The positive ion is named first followed by the negative ion. The oxidation state of the central metal is shown by a Roman numeral in Brackets immediately following the name. The complex negative ends in '-ate'. Thus the name is Tetraamminenickel(II) tetrachloridonickelate (II).

12. The vapour density of N_2O_4 at a certain temperature is 30. Calculate the percentage dissociation of N_2O_4 at this temperature. $N_2O_4(g) \rightleftharpoons 2NO_2(g)$

(A) 9.4% (B) 36.8% (C) 18.7% (D) 53.3%

Ans. **D**

Sol. $x = \frac{D-d}{d} = \frac{(46-30)}{30} = 0.533$

13. How many of following atoms have higher atomic radius than nitrogen (N)? O, Rb, P, C, F, B, Na, K, Cs & Ba
(A) 6 (B) 7 (C) 8 (D) 5

Ans. **C**

Sol. Rb, P, C, B, Na, K, Cs & Ba

14. When freshly precipitated $Fe(OH)_3$ is shaken with small amount of aqueous solution of $FeCl_3$ a colloidal solution is obtained, this is an example of:
(A) Protective action (B) Dissolution (C) Peptization (D) Dialysis

Ans. **C**

Sol. A dark red colored sol of $Fe(OH)_3$ is obtained when freshly prepared precipitate is treated with small amount of $FeCl_3$ solution. The precipitate adsorbs the Fe^{3+} ion from the electrolyte on its surface and develops positive charge on particles. This causes the precipitate to disintegrate into particles of colloidal size.

15. Amongs the following which is wrongly matched?
(A) σ_{1s}; zero nodal plane (B) π_{2p_y} : 1 nodal plane
(C) $\pi^*_{2p_y}$: 2 nodal plane (D) $\sigma^*_{2p_x}$: 2 nodal plane

Ans. **D**

Sol. (4) $\sigma^*_{2p_x}$: 3 nodal plane

16. At 600 K a reaction $S_8(g) \rightleftharpoons 4S_2(g)$ is carried out by taking 2 moles of $S_8(g)$ and 0.2 mole of $S_2(g)$ in a vessel of 1L. Which of the following statement is incorrect? Given $K_c = 6.3 \times 10^{-6}$ M^3 at this temperature. $(R = \frac{1}{12}\frac{atmL}{molK})$
(A) Reaction quotient is 8×10^{-4}
(B) Reaction proceed in backward direction
(C) Reaction proceed in forward direction
(D) $K_p > K_C$

Ans. **C**

Sol. $Q = \frac{[S_2]^4}{[S_8]} = \frac{(0.2)^4}{(2)} = 8 \times 10^{-4}, Q > K_c$

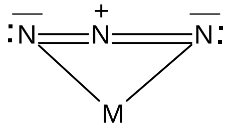
17. Which of the following is correct statement?
(A) For coagulation of aluminium hydroxide sol Ba^{2+} ions is more effective than Na^+
(B) Cellulose solution is an example of macromolecular colloid system.
(C) Sols of metal sulphides are lyophilic.
(D) Schulze-Hardy law states, the bigger the size of the cation, the greater is its coagulating power.

Ans. **B**

18. Which of the following statement is true for azide ion $\left(N_3^-\right)$?
(A) It can act as bidentate ligand.
(B) Two N–N bond lengths are different in the anion.
(C) It is isoelectronic and isostructural with CO_2.
(D) There are two σ and three π bonds.

Ans. **C**

Sol.

(1)  (N_3^-) is linear here four membered ring is not stable.

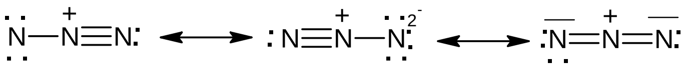
(2) Both N -N bond lengths are identical and that is 1.15 Å

(3) N_3^- and CO_2 both have same number of electrons i.e. 22

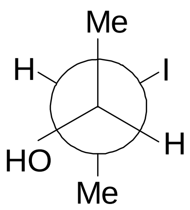
So isoelectronic.

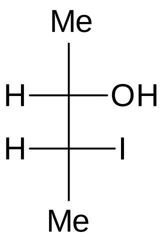
So both are also isostructural

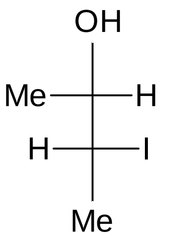
(4) There are two σ and two π bonds.

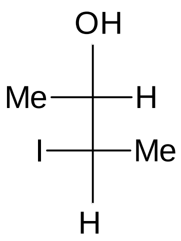


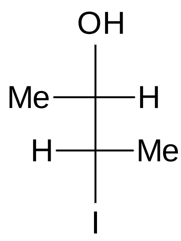
19. Which of the following is the diastereomers of the given compound:



(A) 

(B) 

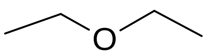
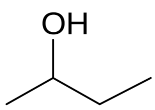
(C) 

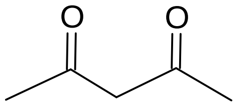
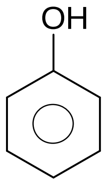
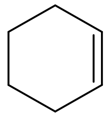
(D) 

Ans. D

Sol. Option (1), (2) , (3)are enantiomer of given compound.

20. How many of the following compounds give benzene on reaction with PhMgBr?

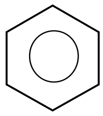
(a)	$CH_3—C\equiv CH$	(b)		(c)	

(d)		(e)		(f)	
(g)	CH_3NH_2	(h)	$CH_3—CH_2—SH$	(i)	$CH_3—C\equiv C—CH_3$

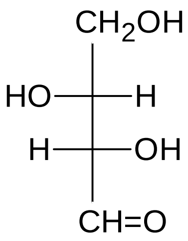
(A) 3　　(B) 5　　(C) 9　　(D) 6

Ans. D

Sol.

PhMgBr + compounds having acidic hydrogen $\left[\begin{matrix} —O—H \\ —N—H \\ —S—H \\ —C\equiv C—H \end{matrix}\right]$ ⟶ 

21. D/L configuration of the following compound is :

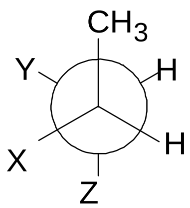


(A) 2D, 3D　　(B) 2L, 3L　　(C) 2D, 3L　　(D) 2L, 3D

Ans. D

Sol.

22. If the newman's projection formula for 2-chlorobutane is given as



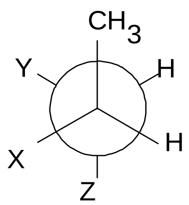
Which of the following is true:

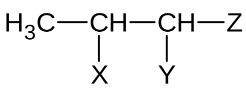
(A) X is Cl and Y & Z are CH_3 & H　　(B) X is CH_3 and Y & Z are H & Cl

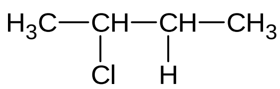
(C) X is H and Y & Z are H & CH_2Cl　　(D) X is CH_3 and Y & Z are CH_3 & Cl

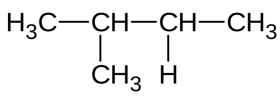
Ans. A

Sol.





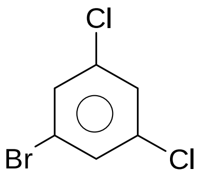
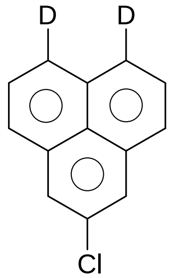
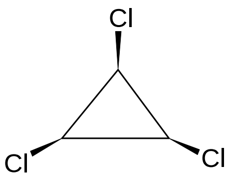
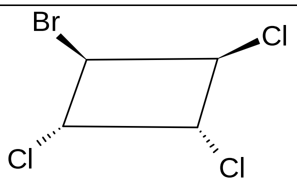
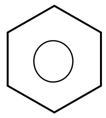
(A) X is Cl and Y & Z are CH_3 & H ⟶ 

(B) X is CH_3 and Y & Z are H & Cl ⟶ 

(C) X is H and Y & Z are H & $CH_2Cl \longrightarrow H_3C—CH(H)—CH(H)—CH_2Cl$

(D) X is CH_3 and Y & Z are CH_3 & Cl $\longrightarrow H_3C—CH(CH_3)—CH(Cl)—CH_3$

23. Which of the following pair of molecules/s show plane of symmetry as well as axis of symmetry (except C_1 axis of symmetry):

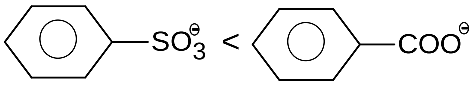
(a)		(b)		(c)	
(d)		(e)			

(A) P, Q, R, T (B) P, Q, R, S (C) Q, R, S, T (D) P, Q, S, T

Ans. A

Sol.

24. The correct order of leaving group ability is :

(A) 

(B) $CF_3SO_3^{\ominus} > CCl_3SO_3^{\ominus}$

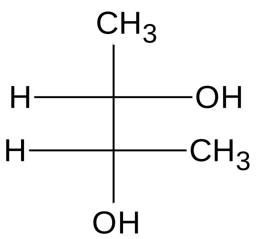
(C) $\overset{\ominus}{C}N > I^{\ominus}$

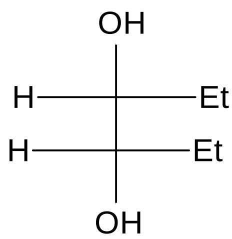
(D) $\overset{\ominus}{N}H_2 > \overset{\ominus}{O}H$

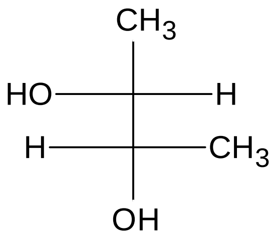
Ans. B

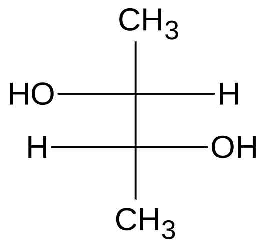
Sol. Leaving group ability $\propto$ stability of anion.

25. Which of the following structure is meso Butane-2, 3-diol ?

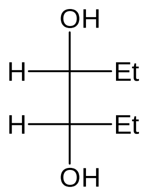
(A) 

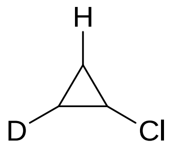
(B) 

(C) 

(D) 

Ans. **C**

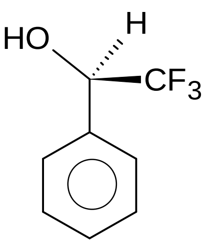
Sol.  It is meso but name is Hexane - 3,4-diol.

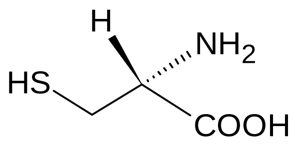
26. Which of the following is correct ?
  (A) All compounds which have chiral carbon are optically active.
  (B) Trans-cyclooctene is more stable than cis-cyclooctene.
  (C) In general bulky substituents on chair conformers prefer to occupy axial positions.
  (D) Number of geometrical isomers of  is two.

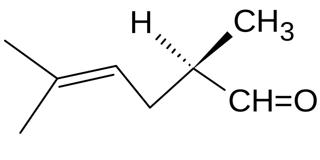
Ans. **D**

Sol. cis-cycloctene have more stable than trans-cylclooctene.

27. The R/S configurations of these compounds are respectively


(I)


(II)


(III)

(A) R, R, R (B) R, S, R (C) R, S, S (D) S, S, S

Ans. **A**

Sol. S → R; II → R; III → R

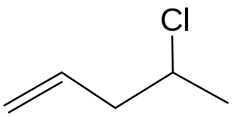
28. Choose the correct matching in the following

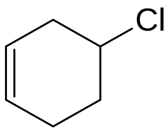
	I	II	III	IV	
(A)	H_2O	$^{\oplus}NH_4$	C_2H_5OH	$H^{\ominus}$	All are Nucleophile.
(B)	$H^{\oplus}$	SO_3	$:CCl_2$	$\overset{\oplus}{N}O_2$	All are Electrophiie.
(C)	$CH_3-CH(OH)-CH_3$	C_2H_5OH	$H-C(=O)-N(CH_3)_2$	$CH_3-S(=O)-CH_3$	All are Polar protic solvent.
(D)	$CH_3-CH(OCH_2CH_3)-CH_3$	CH_3COCH_3	$H-C(=O)-\ddot{N}(CH_3)_2$	H_2O	All are Polar aprotic solvent.

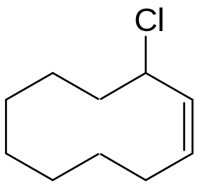
Ans. **B**

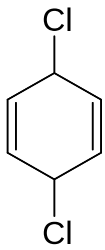
Sol.

29. Which of the following shows optical as well as geometrical isomers

(A) 

(B) 

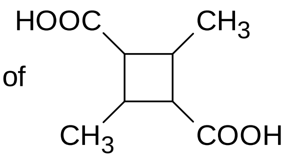
(C) 

(D) 

Ans. **C**

Sol. 2) Min 8C necessary for cycloalkene.

4) POS

30. Total number of optically active stereoisomer of 

is :

(A) 3 (B) 1 (C) 0 (D) 5

Ans. **C**

PART – III (MATHEMATICS)

1. Let $f(x) = ax^7 + bx^3 + cx – 5$, where a, b and c are constants. If f(–7) = 7, then f(7) equals
(A) –17 (B) –7 (C) 14 (D) 21

Ans. **A**

Sol. $f(-x) = -ax^7 - bx^3 - cx - 5$

$\therefore f(x) + f(-x) = -10;$ Put x = 7

$f(7) = -10 - f(-x) = -17$

2. Let S = {1, 2, 3, 4, 5} and A = S × S. A relation R on A is defined as follows: "(a, b) R (c, d) if ad = cb". Then R is
(A) Reflexive but not Symmetric relation. (B) Symmetric but not Reflexive relation.
(C) Transitive but not Reflexive relation. (D) Equivalence relation.

Ans. **D**

Sol. Check for reflexivity:
(a, b) R (a, b) iff ab = ba true
Hence R is reflexive
Check for being symmetric.

$(a,b)R(c,d) \Rightarrow ad = bc$

$(c,d)R(a,b) \Rightarrow cb = da$ true

Check for transitivity

$(a,b)R(c,d) \Rightarrow ad = bc$ ………….(i)

$(c,d)R(e,f) \Rightarrow cf = de$ ………..(ii)

$(a,b)R(e,f) \Rightarrow af = be$ ………….(iii)

Multiple (i) and (ii) we get
Hence R is transitive
So R is an equivalence relation.

3. If x and y are real numbers such that $x^2 + y^2 = 8$, The maximum possible value of x – y, is
(A) 2 (B) 4 (C) $\sqrt{2}$ (D) $\sqrt{2}/2$

Ans. **B**

Sol. $x^2 + y^2 = 8$

$x = 2\sqrt{2}\cos\theta, y = 2\sqrt{2}\sin\theta$

$\therefore x - y = 2\sqrt{2}(\cos\theta - \sin\theta) = 4\cos(\theta + \pi/4)$

$\therefore (x-y)_{max} = 4$

4. Number of point where the function $f(x) = \dfrac{1}{|x|-1} + \tan x$,for $x \in (–3, 3)$ is discontinuous, is
(A) 4 (B) 5 (C) 2 (D) 3

Ans. **A**

Sol. $\dfrac{1}{|x|-1}$ is discontinuous at x = -1, 1

And tanx is continuous at x = -1, 1

Also tanx is discontinuous at $x = -\dfrac{\pi}{2}, \dfrac{\pi}{2}$

And $\dfrac{1}{|x|-1}$ is discontinuous at $x = -\dfrac{\pi}{2}, \dfrac{\pi}{2}$

$\Rightarrow f(x)$ is discontinuous at $x = -\frac{\pi}{2}, \frac{\pi}{2}$

Hence f(x) is discontinuous at four points.

5. Minimum value of $\frac{1}{2}\left(\cot\frac{A}{2} + 3\tan\frac{A}{2}\right)$ where A $\in$ (0, 180^o) occurs when A equals

(A) 60^o (B) 90^o (C) 120^o (D) 150^o

Ans. A

Sol. Using AM $\geq$ GM

$\frac{1}{2}\left(\cot\frac{A}{2} + 3\tan\frac{A}{2}\right) \geq \sqrt{3}$

Hence minimum value is $\sqrt{3}$

now $\cot\frac{A}{2} + 3\tan\frac{A}{2}$

$3\tan^2\frac{A}{2} - 2\sqrt{3}\tan\frac{A}{2} + 1 = 0$

$\tan\frac{A}{2} = \frac{2\sqrt{3} \pm \sqrt{12-12}}{6} = \frac{1}{\sqrt{3}}$

$\frac{A}{2} = 30^0 \Rightarrow A = 60^0$

6. Let k be an integer and p is a prime such that the quadratic equation $x^2 + kx + p = 0$ has two distinct positive integer solutions. The value of (k + p) equals

(A) –1 (B) 0 (C) 3 (D) 6

Ans. A

Sol. Let r_1 and r_2 are two integral solutions

$r_1 + r_2 = -k$ and $r_1r_2 = p$

Since p is prime hence either $r_1 = 1$ or $r_2 = 1$

let $r_1 = 1$; $r_2 = p$

1 + p = - k

$\Rightarrow$ k + p = - 1

7. The value of the expression, $\cos35^o + \cos125^o + 2\sin185^o(\sin130^o + \sin140^o)$ when simplified is

(A) positive and greater than (1/2) (B) negative and less than (–1/2)
(C) 1/2 (D) zero

Ans. D

Sol. $2\cos80^0\cos45^0 - 2\sin5^0(2\sin135^0\cos5^0)$

$2\sin10^0\cos45^0 - 2\sin10^0\sin45^0 = 0$

8. $f(x) = 5^{\sqrt{4x^2-1}}$. How many of the following are true ?

I. The domain of f is $|x| \geq \frac{1}{2}$

II. The range of f(x) is $y \geq 1$

III. The graph of f(x) is symmetric about the y-axis.

IV. f(x) has no critical points.

V. The graph of f(x) never decreases.

(A) one (B) two (C) three (D) four

Ans. C

Sol. only (i), (ii) and (iii) are correct.

f(x) has critical points at $x = \pm\frac{1}{2}$

as $\frac{dy}{dx} \to \infty$ and f(x) is decreasing for x < - ½]

9. If $A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$ then let us define a function f(x) = det. $(A^T A^{-1})$ then which of the following can not be the value of $\underbrace{f(f(f(f..........f(x))))}_{\text{n times}}$ is $(n \geq 2)$

(A) $f^n(x)$ (B) 1 (C) $f^{n-1}(x)$ (D) n f(x)

Ans. D

Sol. $A^T A^{-1} \begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}$

So det. $(A^T A^{-1}) = 1$

So f(x) = 1 is a constant function

10. The equations of L_1 and L_2 are y = mx and n = nx, respectively. Suppose L_1 makes twice as large of an angle with the horizontal (measured counterclockwise from the positive x-axis) as does L_2 and that L_1 has 4 times the slope of L_2. If L_1 is not horizontal, then the value of the product (mn) equals

(A) $\frac{\sqrt{2}}{2}$ (B) $-\frac{\sqrt{2}}{2}$ (C) 2 (D) –2

Ans. C

Sol. Let $m = \tan 2\theta$ and $n = \tan\theta$

And m = 4n

& $m \neq 0$, $\therefore$ $\tan 2\theta \neq 0; \therefore \theta \neq 0$

$\tan 2\theta = 4\tan\theta$

11. Three balls marked 1, 2 and 3 are placed in an urn. One ball is drawn, its number is recorded, then the ball is returned to the urn. This process is repeated and then repeated once more, and each ball is equally likely to be drawn on each occasion. If the sum of the number recorded is 6, the probability that the ball numbered 2 was drawn at all the three occassions, is

(A) $\frac{1}{27}$ (B) $\frac{1}{7}$ (C) $\frac{1}{6}$ (D) $\frac{1}{3}$

Ans. B

Sol. He picked 3 times, the following outcome add up to 6

$$\left.\begin{matrix} 1,2,3 \\ 1,3,2 \\ 2,3,1 \\ 2,1,3 \\ 2,2,2 \\ 3,1,2 \\ 3,2,1 \end{matrix}\right\}$$

All these events are equally likely , ME and exhaustive . Out of 7 cases only one favours

Therefore, there is 1/7 chance.

12. Number of integral values of x the inequality $\log_{10}\left(\frac{2x-2007}{x+1}\right) \le 0$ holds true, is

(A) 1004 (B) 1005 (C) 2007 (D) 2008

Ans. B

Sol. $0 < \frac{2x-2007}{x+1} \le 1$

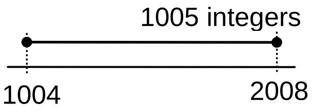
$0 < 2x - 2007 \le x + 1$

$0 < x \le 2008$ ……….(1)

Also $2x - 2007 > 0$

$x > 1003.5$ or $x \ge 1004$ ……(2)

From (1) and (2)



13. Two rays with common end point 'O' forms a 30° angle. Point A lies on one ray, point B on the other ray and AB = 1. The maximum possible length of OB is

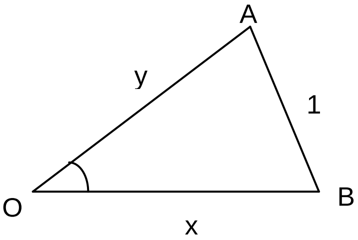
(A) $\sqrt{3}$ (B) $\frac{4}{\sqrt{3}}$ (C) 2 (D) $\frac{\sqrt{3}+1}{\sqrt{2}}$

Ans. C

Sol. Using cosine rule

$1 = x^2 + y^2 - 2xy\cos 30^0$

$\therefore y^2 - xy\sqrt{3} + x^2 - 1 = 0$



As y is real , $D \ge 0$

$3x^2 - 4(x^2 - 1) \ge 0$

$x^2 \le 4 \Rightarrow x \le 2$

Maximum value of x is 2

14. Consider the two functions f(x) = x^2 + 2bx + 1 and g(x) = 2a(x + b), where the variable x and the constants a and b are real numbers. Each such pair of the constants a and b may be considered as a point (a, b) in an ab – plane. Let S be the set of such points (a, b) for which the graphs of y = f(x) and y = g(x) do not intersect (in the xy - plane). The area of S is

(A) 1 (B) π (C) 4 (D) 4π

Ans. B

Sol. We need $x^2 + 2bx + 1 = 2ax + 2ab$ not to have any real solutions, implying that the discriminant is less than or equal to zero. Actually calculating the discriminant and implying, we get $a^2 + b^2 < 1$, which describes a circle of area π in the a – b plane.

15. Let $\Delta_0 = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{32} & a_{32} & a_{33} \end{vmatrix}$ and let Δ_1 denote the determinant formed by the cofactors of elements of Δ_0 and Δ_2 denote the determinant formed by the cofactor at Δ_1 and so on Δ_n denotes the determinant formed by the cofactors at Δ_{n-1} then the determinant value of Δ_n is

(A) Δ_0^{2n} (B) $\Delta_0^{2^n}$ (C) $\Delta_0^{n^2}$ (D) Δ_0^2

Ans. B

Sol. $\Delta_1 = \Delta_0^2$

$\Delta_2 = \Delta_1^2 = \Delta_0^4$

$\Delta_3 = \Delta_2^2 = \Delta_0^8$

$\Delta_4 = \Delta_3^2 = \Delta_0^{16}$

And so on $\Delta_n = \Delta_0^{2^n}$

16. The polynomial $P(x) = x^3 + ax^2 + bx + c$ has the property that the mean of its zeroes, the product of its zeroes, and the sum of its coefficients are all equal. If the y-intercept of the graph of y = P(x) is 2, then the value of b is

(A) –11 (B) –9 (C) 1 (D) 5

Ans. A

Sol. The y-intercept is at x = 0 , so we have c = 2 , meaning that the product of the roots is – 2. We know that a is the sum of the roots . The average of the roots as is equal to the product, so the sum of the roots is -6 and a = 6. Finally 1 + a + b + c = 2 as well, so we have 1 + 6 + b + 2 = -2

$\Rightarrow$ b = - 11

17. If $\sum_{k=10}^{2006} {}^kC_{10}$ simplifies to ${}^nC_p$ where p is prime then (n + p) has the value equal to

(A) 2017 (B) 2018 (C) 2019 (D) 2020

Ans. B

Sol. $S = {}^{10}C_{10} + {}^{11}C_{10} + {}^{12}C_{10} + {}^{13}C_{10} + ... + {}^{2006}C_{10}$

$= \underbrace{{}^{11}C_0 + {}^{11}C_1} + {}^{11}C_2 + {}^{13}C_3 + ... + {}^{2006}C_{1996}$

$= \underbrace{{}^{12}C_1 + {}^{12}C_2} + ...$

${}^{13}C_2$ and so on

$\therefore S = {}^{2007}C_{1996} = {}^{2007}C_{11} = {}^nC_p$

$\therefore n + p = 2018$

18. The number of triplets of positive integers {a, b, c} such that a × b × c = 2310, is

(A) 40 (B) 30 (C) 25 (D) 15

Ans. A

Sol. N = 2310 = 2 · 3 · 5 · 7 · 11

We want to either partition these 5 prime numbers in groups of 2 with the other third number 1 or groups of 3.

$5 = 1 + 4 + = 2 + 3$ and 5 = 1 +1 + 3 = 1 + 2 + 2

Hence , number of ways

$\frac{5!}{1!4!} + \frac{5!}{2!3!} + \frac{5!}{1!1!2!3!} + \frac{5!}{1!2!2!2!}$

$= 5 + 10 + 10 + 15 = 40$

19. $\lim_{x\to 0} \frac{\sqrt[3]{1+3x} - 1 - x}{(1+x)^{101} - 1 - 101x}$ has the value equal to

(A) $-\dfrac{3}{5050}$ (B) $-\dfrac{1}{5050}$ (C) $\dfrac{1}{5051}$ (D) $\dfrac{1}{4950}$

Ans. **B**

Sol. Use L' Hospital's Rule twice.

20. Number of real solution of equation $16\sin^{-1}x\ \tan^{-1}x\ \text{cosec}^{-1}x = \pi^3$ is/are

(A) 0 (B) 1 (C) 2 (D) infinite

Ans. **B**

Sol. Domain is x = ± 1, however only x = 1 satisfies

21. $\lim_{x\to 0}\dfrac{(2^{\sin x}-1).\ln(1+\sin 2x)}{x(\text{arc}\ \tan x)}$ equals

(A) ln 4 (B) ln 2 (C) $(\ln 2)^2$ (D) 1

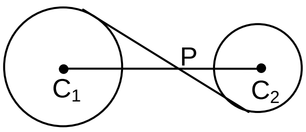
Ans. **A**

Sol. $\lim_{x\to 0}\dfrac{(2^{\sin x}-1)\sin x}{\sin x}\cdot\dfrac{\ln(1+\sin 2x)}{x(\tan^{-1}x)}$

$= (\ln 2)\lim_{x\to 0}\dfrac{\sin x}{\tan^{-1}x}\cdot\lim_{x\to 0}\ln(1+\sin 2x)^{\frac{1}{x}}$

$= (\ln 2)(1)\lim_{x\to 0}\dfrac{(1+\sin 2x-1)}{x} = 2\ln 2 = \ln 4$

22. In the figure given,

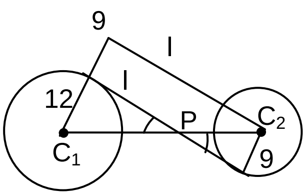


Two circles with centres C_1 and C_2 are 35 units apart, i.e. $C_1C_2 = 35$. The radii of the circles with centres C_1 and C_2 are 12 and 9 respectively. If P is the intersection of C_1C_2 and a common internal tangent to the circles, the length C_1P equals

(A) 18 (B) 20 (C) 12 (D) 15

Ans. **B**

Sol.



Let $C_1P = a$

$PC_2 = 35 - a$

Hence, $\dfrac{9}{35-a} = \dfrac{12}{a} \Rightarrow 3a = 4(35-a)$

$\Rightarrow 7a = 4\cdot 35 \Rightarrow a = 20$

Integer Value Correct Type

23. If in 6 trials, X is a binomial variate which follows the relation 9P(X = 4) = P(X = 2), then what is the reciprocal of probability of success?

Ans. **4**

Sol. $9P(x=4) = P(x=2)$

$9\dfrac{6!}{4!2!}(P)^4(1-P)^2 = \dfrac{6!}{4!2!}P^2(1-P)^4$

$P^2(1-P)^2[9P^2-(1-P)^2]=0$

$P=-\frac{1}{2}$ Not possible

$P=\frac{1}{4}$

24. If $\sum_{i=1}^{9}(x_i-8)=9$ and $\sum_{i=1}^{9}(x_i-8)^2=45$ then find standard deviation of $x_1, x_2, \ldots, x_9$

Ans. 2

Sol. $S.D(x_i)=S.D.(x_i-8)$

$\sqrt{\frac{\sum(x_i-8)^2}{n}-\left(\frac{\sum(x_i-8)}{n}\right)^2}$

$=\sqrt{\frac{45}{9}-1}=2$

25. The value of $\frac{1}{5}(\sec^2(\tan^{-1}3)+\operatorname{cosec}^2(\cot^{-1}2))$ is equal to

Ans. 3

Sol. We have $\sec^2(\tan^{-1}3)+\operatorname{cosec}^2(\cot^{-1}2)$

$=(\sec(\tan^{-1}3))^2+(\operatorname{cosec}(\operatorname{cosec}^{-1}\sqrt{5}))^2$

$=(\sqrt{10})^2+(\sqrt{5})^2=15$

26. Let $A=\begin{bmatrix}1 & 4\\ 2 & 3\end{bmatrix}$ and $A^4-4A^3-5A^2+2I=\lambda I$ (is the unit matrix of order 2), then find λ.

Ans. 2

Sol. Characteristic equation f A is

$\lambda^2-4\lambda-5=0$

$\therefore A^2-4A-5I=0$ (By Hamilton Theorem)

$\therefore E=A^4-4A^3-5A^2+2I$

$=A^2(A^2-4A-5I)+2I$

$=0+2I=2I$

$\therefore \lambda=2$

27. Let $f(x)=\begin{cases}\frac{1+a\cos 2x+b\cos 4x}{x^2\sin^2 x} & \text{if } x\neq 0\\ c & \text{if } x=0\end{cases}$. If f (x) is continuous at x = 0, then find the value of (b + c – 3a).

Ans. 7

Sol. $\lim_{x\to 0}\frac{1+a\cos 2x+b\cos 4x}{x^4}$

as $x\to 0$, $N^r\to 1+a+b$

$D^r\to 0$

for existence of limit a + b + 1 = 0

$\therefore c=\lim_{x\to 0}\frac{a\cos 2x+b\cos 4x-(a+b)}{x^4}$ ……..(2)

$$= -\lim_{x\to 0} \frac{\frac{a(1-\cos 2x)}{x^2} + \frac{b(1-\cos 4x)}{x^2}}{x^2}$$

Limit of N^r $4a\left(\frac{1}{2}\right) + 16b\left(\frac{1}{2}\right)$

$\Rightarrow 2a + 8b = 0 \Rightarrow a = -4b$

Hence $-4b + b = -1 \Rightarrow b = \frac{1}{3}$ and $a = -\frac{4}{3}$

Hence $c = \lim_{x\to 0} \frac{4(1-\cos 2x) - (1-\cos 4x)}{3x^4}$

$= \frac{8\sin^2 x - 2\sin^2 2x}{3x^4}$

$\frac{8}{3} \cdot \frac{\sin^2 x}{x^2} \cdot \frac{\sin^2 x}{x^2} = \frac{8}{3}$

28. The remainder when $4^{87} + 6^{87} + 32$ is divided by 25, is

Ans. 2

Sol. $4^{87} + 6^{87} + 32$

$(5-1)^{87} + (5+1)^{87} + 32$

$= (1+5)^{87} - (1-5)^{87}$

$= 2[{}^{87}C_1 \cdot 5 + {}^{87}C_3 \cdot 5^3 + ... + {}^{87}C_{87} \cdot 5^{87}] + 32$

$= 10 \cdot 87 + 2[{}^{87}C_3 \cdot 5^3 + ... + {}^{87}C_{87} \cdot 5^{87}] + 32$

= 902 + an expression divisible by 25

$\Rightarrow \frac{902}{25} = 36 + \frac{2}{25}$

$\Rightarrow$ remainder is 2 .

29. Let $A = \begin{bmatrix} 2 & 2 & 1 \\ 2 & 5 & 2 \\ 1 & 2 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -x & -y & z \\ 0 & y & 2z \\ x & -y & z \end{bmatrix}$ where x, y, z $\in$ R. If $B^T AB = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 27 & 0 \\ 0 & 0 & 42 \end{bmatrix}$ then the number of ordered triplet (x, y, z) is

Ans. 8

Sol. $AB = \begin{bmatrix} -x & -y & 7z \\ 0 & y & 14z \\ x & -y & 7z \end{bmatrix}$

Equating coefficients $x^2 = 4; y^2 = 9$ and $z^2 = 1$

$\Rightarrow x = \pm 2, y = \pm 3$ and $z = \pm 1$

Number of solution $= 2 \times 2 \times 2 = 8$

30. Let ℓ_1 be the line 4x + 3y = 3 and ℓ_2 be the line y = 8x. L_1 is the line formed by reflecting ℓ_1 across the line y = x and L_2 is the line formed be reflecting ℓ_2 across the x-axis. If θ is the acute angle between L_1 and L_2 such that $\tan\theta = \frac{a}{b}$, where a and b are co-prime then find (a – b)

Ans. 1

Sol. $\ell_1 : 4x + 3y = 3$

$f(x) = y = \dfrac{3 = 4x}{3}$ **…….(1)**

Since f(x) and $f^{-1}(x)$ are the mirror images of each other in
The line y = x hence we find $f^{-1}(x)$

Now $y = f(x) \Rightarrow f^{-1}(y) = x$

$\therefore f^{-1}(x) = \dfrac{3(1-y)}{4}$

$4y = 3 - 3x$

$L_1 = 3x + 4y - 3 = 0$

$m_1 = 3/4$

$L_2 = y = -8x$ with $m_2 = -8$

If θ is the acute angle between the lines

$\tan\theta = \left|\dfrac{m_2 - m_1}{1 + m_1 m_2}\right|$

$= \left|\dfrac{-8 + \frac{3}{4}}{1 + (-8)\left(-\frac{3}{4}\right)}\right| = \left|\dfrac{-29}{28}\right| = \dfrac{29}{28}$

$\Rightarrow$ a = 29 and b = 28

$\therefore a - b = 29 - 28 = 1$